GRAPHS AND THE SOLUTION OF EQUATIONS

Transformations on curves

Key Points

1 The table below shows the different **effects** transformations have graphically on curves and algebraically on their corresponding equations.

Transformation	Graphical effect on curve		Algebraic effect on $y = f(x)$	
	Coordinates	Curve	Action	Result
Translation k units along the +ve x-axis	Adds k to each x-coordinate	Shifts the curve k units along the +ve x-axis	Replace each x term with $x - k$	$y = f(x - k)$
Dilate along the +ve x-axis by a factor of k	Each x-coordinate is multiplied by a factor of k	Stretches curve by a factor of k horizontally	Replace each x term with $\frac{x}{k}$	$y = f\left(\frac{x}{k}\right)$
Reflection about the y-axis	Each x-coordinate is multiplied by -1	Reflects curve about the y-axis	Replace each x term with $-x$	$y = f(-x)$
Translation k units along the +ve y-axis	Adds k to each y-coordinate	Shifts the curve k units along the +ve y-axis	Adds k to $f(x)$	$y = f(x) + k$
Dilate along the +ve y-axis by a factor of k	Each y-coordinate is multiplied by a factor of k	Stretches curve by a factor of k vertically	$f(x)$ is multiplied by k	$y = kf(x)$
Reflection about the x-axis	Each y-coordinate is multiplied by -1	Reflects curve about the x-axis	$f(x)$ is multiplied by -1	$y = -f(x)$

2 When determining the transformations required to achieve a given result, consideration must be given to the **order** in which the transformations are applied.
For example, for $y = -kf(-ax + b) + m$

- where $f(-ax + b)$ is left in this form **(expanded form)**:
  - **T** Translate b units left along the x-axis — $f(x) \rightarrow f(x + b)$
  - **D** Dilate along the x-axis by factor $\frac{1}{a}$ — $f(x + b) \rightarrow f(ax + b)$
  - **R** Reflect about the y-axis — $f(ax + b) \rightarrow f(-ax + b)$
  - **R** Reflect about the x-axis — $f(-ax + b) \rightarrow -f(-ax + b)$
  - **D** Dilate along the y-axis by factor k — $-f(-ax + b) \rightarrow -kf(-ax + b)$
  - **T** Translate m units up along the y-axis — $-kf(-ax + b) \rightarrow -kf(-ax + b) + m$
- where $f(-ax + b)$ is rewritten as $f\left(-a\left(x - \frac{b}{a}\right)\right)$ **(factored form)**:
  - **R** Reflect about the y-axis — $f(x) \rightarrow f(-x)$
  - **D** Dilate along the x-axis by factor $\frac{1}{a}$ — $f(-x) \rightarrow f(-ax)$
  - **T** Translate $\frac{b}{a}$ units right along the x-axis — $f(-ax) \rightarrow f\left(-a\left(x - \frac{b}{a}\right)\right)$
  - **R** Reflect about the x-axis — $f(-ax + b) \rightarrow --f(-ax + b)$
  - **D** Dilate along the y-axis by factor k — $-f(-ax + b) \rightarrow -kf(-ax + b)$
  - **T** Translate m units up along the y-axis — $-kf(-ax + b) \rightarrow -kf(-ax + b) + m$

CHECKLIST – Can you:
1. Differentiate its effect on the curve (coordinates) and on its equation for each transformation?
2. Determine the transformations (and its order of application) required for a desired result?